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TOPICAL ISSUE

Terminal Control of Center of Mass Motion and Propellant Consumption in Liquid-Propellant Rocket Carriers

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Abstract—We dedicate this work to the memory of academician B.N. Petrov. It develops the principles of terminal control of rocket carriers formulated by him. Next-generation rocket carriers implement the principle of interconnected, coordinated terminal control of the center of mass motion and propellant consumption. In this article we consider the problem of synthesizing such control and the main principles of its implementation.

Keywords: terminal control, predictive model

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1. INTRODUCTION

The beginning of B.N. Petrov's creative activity coincided with the time when our war-exhausted country made a gigantic breakthrough, opening the way to space for humanity. Soviet science played an important role in this breakthrough. Many of the problems related to the creation of rocket carriers belong to automatic control of mobile objects. B.N. Petrov's profound knowledge in this field and his erudition allowed him to actively participate in the development of new unique automatic control problems and in the development and discussion of our country's space programs alongside leading figures in rocket and space science and technology.

He rightfully became one of the founders of domestic cosmonautics, working for many years in close contact with S.P. Korolev, V.P. Glushko, M.K. Yangel, V.N. Chelomey, V.F. Utkin, and N.A. Pilyugin.

The results of B.N. Petrov and Institute of Dynamics Research, which he headed in the development of methods of modeling and regulating liquid rocket engine thrust and propellant component ratio, are used in many onboard terminal systems. These systems significantly increase the energy of rockets by dramatically reducing the guaranteed propellant reserves. The book by Chertok "Rockets and People" [1] notes the significance of this work.

Understanding the specifics of onboard terminal systems and the peculiarities of organizing control processes allowed B.N. Petrov and his students to classify these systems as a separate class among other automatic control systems. The monograph "Onboard Terminal Control Systems" [2] develops the principles and elements of the theory of this class of systems.

The ideas of B.N. Petrov have further evolved and been applied in modern developments of the Institute in the field of rocket and space technology, resulting in the creation of terminal control systems for new-generation rocket carriers and booster blocks for space and defense purposes (upgraded carrier rockets "Soyuz-2", "Angara" rocket family, "Sarmat" rocket, rocket boosters under development "Soyuz-5", "Amur", and the KVTK booster block).

Next-generation rocket carriers implement the principle of interconnected, coordinated terminal control of the center of mass motion and propellant consumption. In this article we consider the problem of synthesizing such control and the main principles of its implementation.

2. PROBLEM STATEMENT

Consider the control of the center of mass motion of the rocket carrier in the exoatmospheric flight phase.

To simplify, we assume the following:

—Aerodynamic forces are absent,

—The Earth's gravitational field is parallel to the surface and the acceleration of the gravitational force is constant at all altitudes $\vec{g} = \text{const}$.

—Rotation of Earth is neglected.

The motion of the center of mass of the rocket stage in the longitudinal plane (the plane of the trajectory) in the exoatmospheric flight phase is described by the following equations:

$$\begin{cases} \dot{V}_x = \dfrac{P}{m_\kappa + m}\cos(\vartheta), \quad \dot{V}_y = \dfrac{P}{m_\kappa + m}\sin(\vartheta) - g, \quad P = wr, \\ \dot{x} = V_x, \quad \dot{y} = V_y, \quad \dot{m} = -r, \\ \dot{\vartheta} = \omega, \\ \dot{\omega} = \varphi(\vartheta, \omega, \vartheta_{\text{des}}), \end{cases} \tag{1}$$

where x, y are horizontal and vertical coordinates, m is propellant mass, m_κ is dry mass of the stage, r is propellant consumption rate, w is specific exhaust velocity, P is engine thrust, g is acceleration of gravity, ϑ is pitch angle, ϑ_{des} is control input (desired value of ϑ) for changing the pitch angle, V_x, V_y are horizontal and vertical velocity components.

The equation for the pitch angle ϑ and the angular velocity ω in (1) simplistically describes the operation of the stabilization system.

Coordinates x, y, m, ϑ, and their derivatives are functions of time t, $t \in [t_0, t_k]$, t_k is the terminal time.

Note that the pitch angle ϑ converges to the value $\vartheta_{\text{des}}(t)$ in a significantly shorter time than t_k.

For the final stage, reaching the specified altitude with zero vertical velocity is required:

$$\begin{cases} y(t_k) = y_k, \\ V_y(t_k) = 0. \end{cases} \tag{2}$$

No conditions are set for the horizontal velocity component. Solving the problem assumes maximizing the horizontal component.

For the lower stages of the rocket, we state the problem of hitting the designated burned-out stage impact areas. In this case, we can determine boundary conditions for deviation of the flight range L of the burned-out stage due to deviations of the motion coordinates at the end of the flight from the target values:

$$\delta L = \zeta_x(x(t_k) - x_k) + \zeta_y(y(t_k) - y_k) + \zeta_{Vx}(V_x(t_k) - V_{xk}) + \zeta_{Vy}(V_y(t_k) - V_{yk}) = 0, \tag{3}$$

where ζ_x, ζ_y, ζ_{Vx}, ζ_{Vy} are partial derivatives of δL with respect to the motion coordinates, δ is deviation of the flight range from the target value.

We can write the equations determining the apparent velocity change and the engine propellant consumption processes in the following form:

$$\begin{aligned} &\dot{W} = \frac{rg}{m_\kappa + m} P_{sp}, \quad P_{sp} = \frac{w}{g}, \quad m = m_o + m_f, \quad r = r_o + r_f, \\ &\dot{m}_o = -r_o, \quad \dot{m}_f = -r_f, \quad K_m = \frac{\dot{m}_o}{\dot{m}_f}, \quad P_{sp} = \varphi(K_m), \\ &\dot{r}_o = f_o(r_o, \alpha_{K_m}, \alpha_R), \quad \dot{r}_f = f_f(r_f, \alpha_{K_m}, \alpha_R), \end{aligned} \tag{4}$$

with initial conditions accounting for fueling errors and pre-launch propellant component consumption scatter at the moment of the fuel consumption control system activation $m_o(t_0)$, $m_f(t_0)$.

Here, m_o, m_f are the oxidizer and fuel masses, P_{sp} is the specific thrust of the propulsion system, r_o, r_f are the propellant consumption rates determined by the engine equations, α_{K_m}, α_R are the positions of the engine control devices determined by the specified values of the propellant component consumption ratio coefficient K_m and thrust engine condition R.

All coordinates W, m_o, m_f, r_o, r_f and their derivatives are functions of time. We consider them on a bounded time interval t, $t \in [t_0, t_k]$, t_k is terminal time.

The positions of the engine control devices that result in the desired values of the fuel component consumption ratio coefficient $K_m(t)$ and the thrust mode $R(t)$ for engine operation are determined by static nonlinear engine equations:

$$\alpha_{K_m}(t) = f_{K_m}(K_m(t), R(t)), \quad \alpha_R(t) = f_R(K_m(t), R(t)), \quad R(t) = \frac{P(t)}{P_{\text{nom}}}(t).$$

We assume here that $K_m(t)$ is calculated in the algorithm of the terminal control system for object (3), and $R(t)$ is determined by the specified thrust program.

Note that the transients of the propellant consumption rate r_o, r_f in response to the position change of the engine control devices $\alpha_{K_m}(t)$, $\alpha_R(t)$ conclude in a time significantly shorter than t_k.

We impose constraints on the value of the fuel component consumption ratio coefficient that can change during control. We determine the boundary values based on the conditions for stable engine operation and significantly depend on the thrust mode: $K_{m\,\min}(R, t) \leqslant K_m(t) \leqslant K_{m\,\max}(R, t)$.

In this case, we impose final terminal conditions on the remaining propellant components at the moment of engine shutdown and determine them based on the requirements for safe engine shutdown. We specify the conditions as inequalities meaning the necessity of positive values of the remaining propellant components at the moment of engine shutdown, generated by the control system, relative to the propellant level that ensures a safe engine shutdown:

$$m_o(t_k) - m_{o\,\min} > 0, \quad m_f(t_k) - m_{f\min} > 0. \tag{5}$$

Here $m_{o\,\min}$, $m_{f\min}$ are the remaining propellant components that are not spent due to the intake design features and accounting for the control system errors.

We include the values $m_{o\,\min}$, $m_{f\min}$ in m_κ. We understand $m(t)$, $m_o(t)$, $m_f(t)$ as the values of the mass of the current propellant component excluding $m_{o\,\min}$, $m_{f\min}$.

Let's define the vector of residuals of the specified boundary conditions (2), (3), (5) for the terminal problem solution, and the vector of control inputs:

$$\begin{aligned} &z_0 = (y(t_k) - y_k, V_y(t_k), m_o(t_k), m_f(t_k)) \text{ — for the terminal stage}, \\ &z_0 = (\delta L, m_o(t_k), m_f(t_k)) \text{ — for the bottom stages}, \\ &u = (\vartheta_{\text{des}}, K_m, t_k). \end{aligned} \tag{6}$$

Note that the value of R, which determines the engine thrust program, is a specified function of time and is not included in the vector of control inputs u. The terminal time moment t_k can vary and can be used as a control parameter to solve the terminal problem.

The main objective of terminal control is to minimize the residuals of the boundary conditions. In addition to satisfying the boundary conditions, terminal systems also have other requirements, the physical content of which can include energy resource costs, time costs, and control losses. In this work, we limit the problem of criterion synthesis to boundary conditions, the fulfillment of which is a priority.

The control object of the considered terminal system, in terms of transition to the specified final state, is quite inertial (it represents integrating elements).

We achieve control of these processes by influencing other coordinates of the object ϑ, r_o, r_f with rapidly decaying dynamics of their transients. The essence of such control lies in setting the desired steady-state values of these coordinates.

Control of the coordinates ϑ, r_o, r_f (by changing the positions of actuators, drives, fins, etc.) consists of stabilizing these coordinates of the object relative to the specified values determined by the vector $u(t)$. The operation of the closed stabilization loop is simplified by a system of equations for ϑ, r_o, r_f.

In this case, we consider the operation of the stabilization loop in terms of transient responses to changes in the control input. We assume that the transient process is completed in an interval significantly shorter than the terminal control interval.

3. CONTROL ALGORITHM SYNTHESIS IN THE CLASS OF PIECEWISE-CONSTANT FUNCTIONS OF PREDICTED RESIDUALS OF THE TERMINAL CONDITIONS

Let us consider the object terminal control problem (1), (4) within the class of predictive model systems.

Let us integrate (1) on prediction interval $\tau \in [t, t_{\text{command}}]$, where t_{command} is the predicted value of the terminal time moment. We define the current initial rocket center of mass coordinates x, y, V_x, V_y at time t in the inertial navigation system. We substitute propellant mass $m(t)$ equation in (1) with $m_{\text{mod}}(t)$ formed in the propellant management algorithm:

$$\begin{aligned} \dot{m}_{\text{mod}}(t) &= r_{\text{mod}}(t), \\ r_{\text{mod}}(t) &= r_{\text{cycl}}(t)(1+\lambda(t)), \end{aligned} \tag{7}$$

where r_{cycl} is cumulative propellant consumption corresponding to a given cyclogram of the engine's thrust operation mode, $\lambda(t)$ is controlled parameter of the model that corrects $r_{\text{cycl}}(t)$ in the propellant consumption model. The physical analog of $\lambda(t)$ is the relative deviation of the cumulative consumption from its nominal value.

Note that the cumulative propellant consumption value corresponding to a given cyclogram of the engine's thrust operation mode ($r_{\text{cycl}}(t)$) can be determined based on measurements of apparent acceleration and equation for $\dot{W}$ in (4).

We integrate (1), (7) in the interval $\tau \in [t, t_{\text{command}}]$ with the assumption that $\vartheta(\tau) = \vartheta(t)$, $r(\tau) = r_{\text{cycl}}(\tau)(1+\lambda(t))$, $m(t) = m_{\text{mod}}(t)$.

Let us define t_{command} from condition $m_{\text{mod}}(t) - \int_t^{t_{\text{command}}} r_{\text{mod}}(\tau)d\tau = 0$.

Let us define the values of the predicted residuals $y(t_{\text{command}}) - y_k$, $V_y(t_{\text{command}})$, $\delta L(t_{\text{command}})$.

When integrating (1) we can use the integral expressions presented in [3].

We take the time moment t such that $m_{\text{mod}}(t) = 0$ is the value of the terminal time moment t_k (engine shutdown).

Regarding the management of propellant components, the predictive model includes equation (7) and equations of the processes of change of the mass of propellant components (4). Taking into account the interdependence of equation (7) with (4), let us define the equations for the deviations of the current values of oxidizer and fuel masses from the model analogues, formed from the model value of the total propellant mass according to the nominal value of ratio coefficient K_m:

$$\begin{aligned} \Delta m_{\text{o}}(t) &= m_{\text{o}}(t) - m_{\text{mod}}(t)\frac{K_{m\,\text{nom}}}{K_{m\,\text{nom}}+1}, \\ \Delta m_{\text{f}}(t) &= m_{\text{f}}(t) - m_{\text{mod}}(t)\frac{1}{K_{m\,\text{nom}}+1}, \end{aligned} \tag{8}$$

where $m_{\text{o}}(t)$, $m_{\text{f}}(t)$ are determined based on measurements of discrete level sensors in tanks.

For the deviations (8), we can obtain equations of the following form:

$$\begin{aligned} \Delta \dot{m}_{\text{o}}(t) &= r_{\text{o}}(t) - r_{\text{mod}}(t)\frac{K_{m\,\text{nom}}}{K_{m\,\text{nom}}+1}, \\ \Delta \dot{m}_{\text{f}}(t) &= r_{\text{f}}(t) - r_{\text{mod}}(t)\frac{1}{K_{m\,\text{nom}}+1}. \end{aligned} \tag{9}$$

Let us integrate equations (9) in interval $\tau \in [t, t_{\text{command}}]$ assuming $r_{\text{o}}(\tau) = r_{\text{o}}(t)$, $r_{\text{f}}(\tau) = r_{\text{f}}(t)$, $r_{\text{mod}}(\tau) = r_{\text{cycl}}(\tau)(1+\lambda(t))$, and initial conditions $\Delta m_{\text{o}}(t)$, $\Delta m_{\text{f}}(t)$.

Determine the values of the predicted residuals $\Delta m_{\text{o}}(t_{\text{command}})$, $\Delta m_{\text{f}}(t_{\text{command}})$.

Due to predictive model of the object (1), (4), the vector of predicted boundary condition residuals (6) is defined as

$$\begin{aligned} z(t) &= (y_{pr}(t_{\text{command}}) - y_k, V_{ypr}(t_{\text{command}}), \Delta m_o(t_{\text{command}}), \Delta m_f(t_{\text{command}})) \\ &\qquad \text{— for the terminal rocket stage,} \\ z(t) &= (\delta L, \Delta m_o(t_{\text{command}}), m_f(t_{\text{command}})) \text{ — for the bottom rocket stage,} \end{aligned} \tag{10}$$

and the vector of control inputs in form $u = (\vartheta_{\text{des}}, K_m, \lambda)$.

If $t \to t_k$, $t_{\text{command}} \to t_k$, $z(t) \to z_0$.

We solve the problem of terminal control of object (1), (4) by forming feedback control based on predicted boundary condition residuals (10).

Let $x_T(t) = (x(t), y(t), V_x(t), V_y(t), \Delta m_o(t), \Delta m_f(t), m_{\text{mod}}(t))$ be the vector of coordinates of the predicted model of the object (1), (7), (9) supplemented with equations for $\dot{m}_o, \dot{m}_f, \dot{r}_o, \dot{r}_f$, which determine the boundary condition residuals, and let $x_u(t) = (\vartheta(t), r_o(t), r_f(t), \lambda(t))$ be the vector of coordinates directly influenced by the control inputs.

As shown in [4, 5], we determine the derivative with respect to time and the differential equation for the vector of predicted boundary condition residuals $z(t)$ by differentiating $z(t)$ as a composite function:

$$\frac{dz(t)}{dt} = \frac{\partial z(t)}{\partial x_T(t_{\text{command}})}\left[\frac{\partial x_T(t_{\text{command}})}{\partial x_u(t)}\frac{dx_u(t)}{d(t)} + \frac{dt_{\text{command}}}{dt}\frac{dx_T(t_{\text{command}})}{d(t)}\right].$$

We choose control inputs $\vartheta_{\text{des}}(t)$, $K_m(t)$, $\lambda(t)$ from the class of piecewise-constant functions of time. The control input for the pitch angle ϑ_{des} changes discretely at moments in time when the information is updated from the inertial navigation system. The control inputs $K_m(t)$ and $\lambda(t)$

for the fuel consumption processes change at discrete moments in time when the levels of the components in the tanks are measured. At these same moments transient processes for $r_o(t)$, $r_f(t)$ appear and the quantities $r_{\text{mod}}(t)$ and $t_{\text{command}}(t)$ change abruptly.

For piecewise-constant control, we can obtain the difference equations for $z(t)$ from the differential equations. We introduce notation for the components of the residuals vector:

$$z_y(t) = y_{pr}(t_{\text{command}}) - y_k, \quad z_V(t) = V_{ypr}(t_{\text{command}}),$$
$$z_{m_o}(t) = \Delta m_o(t_{\text{command}}), \quad z_{m_f}(t) = \Delta m_f(t_{\text{command}}), \quad z_\delta(t) = \delta(t_{\text{command}}).$$

We can express the difference equations for the components of the vector $z(t)$ as follows. In terms of controlling the motion of the center of mass, the difference equations are determined for discrete moments in time t_i when the navigation information is updated $i = 0, 1, 2, \ldots, I$, $t_{I+1} = t_k$ (when $\lambda(t) = \text{const}$, $t_{\text{command}}(t) = \text{const}$):

$$\begin{aligned}
z_y(t_{i+1}) &= z_y(t_i) + \frac{\partial z_y}{\partial \vartheta}(t_i)\Delta\vartheta_i,\\
z_{Vy}(t_{i+1}) &= z_{Vy}(t_i) + \frac{\partial z_{Vy}}{\partial \vartheta}(t_i)\Delta\vartheta_i,\\
z_\delta(t_{i+1}) &= z_\delta(t_i) + \frac{\partial z_\delta}{\partial \vartheta}(t_i)\Delta\vartheta.
\end{aligned} \tag{11}$$

Here

$$\Delta\vartheta_i = \int\limits_{t_i}^{t_i+\delta t} \dot\vartheta(\tau)d\tau,$$

where δt is time interval of the transient in object (1) with respect to coordinate ϑ during an abrupt control input ϑ_{des} change at time momentt_i.

Furthermore, at time moments t_j of discrete measurement of the propellant level in tanks, the aforementioned residuals change due to changes in $\lambda(t)$, $t_{\text{command}}(t)$.

Let us assume that the level sensors conduct discrete measurements at one of the discrete time moments of navigational information update $t_j = t_i$. Let us add terms accounting for abrupt changes of $\lambda(t)$ and $t_{\text{command}}(t)$ to (11):

$$\begin{aligned}
z_y(t_{i+1}) &= z_y(t_i) + \frac{\partial z_y}{\partial \vartheta}(t_i)\Delta\vartheta_i + \frac{\partial z_y}{\partial r_{\text{mod}}}(t_i) r_{\text{cycl}}(t_j)\Delta\lambda_j + \Delta t_{\text{command}j}\dot y(t_{\text{command}}),\\
z_{V_y}(t_{i+1}) &= z_{V_y}(t_i) + \frac{\partial z_{V_y}}{\partial \vartheta}(t_i)\Delta\vartheta_i + \frac{\partial z_{V_y}}{\partial r_{\text{mod}}}(t_i) r_{\text{cycl}}(t_j)\Delta\lambda_j + \Delta t_{\text{command}j}\dot V_y(t_{\text{command}}),\\
z_\delta(t_{i+1}) &= z_\delta(t_i) + \frac{\partial z_\delta}{\partial \vartheta}(t_i)\Delta\vartheta_i + \frac{\partial z_\delta}{\partial r_{\text{mod}}}(t_i) r_{\text{cycl}}(t_j)\Delta\lambda_j\\
&\quad + \Delta t_{\text{command}j}(\zeta_x \dot x(t_{\text{command}}) + \zeta_y \dot y(t_{\text{command}}) + \zeta_{Vx}\dot V_x(t_{\text{command}}) + \zeta_{Vy}\dot V_y(t_{\text{command}})).
\end{aligned} \tag{12}$$

Here $\Delta t_{\text{command}j}$ is difference of $t_{\text{command}j}$ values determined from equation (7) at t_j while $\lambda = \lambda(t_j)$ and $\lambda = \lambda(t_j) + \Delta\lambda_j$. We can determine the value of this difference with the following approximate expression: $\Delta t_{\text{command}j} = \zeta_{tk}(t_j)\Delta\lambda_j$.

In regard to propellant management, we define difference equations for discrete time moments t_j of information update of the level sensors:

$$\begin{aligned} z_{m_o}(t_{j+1}) &= z_{m_o}(t_j) + \frac{\partial z_{m_o}}{\partial r_o}(t_j)\Delta r_{oj} + \frac{\partial z_{m_o}}{\partial r_{\text{mod}}}(t_j) r_{\text{cycl}}(t_j)\Delta\lambda_j \\ &\quad + (r_o(t_j) - r_{\text{mod}}(t_j)\frac{K_{m\,\text{nom}}}{K_{m\,\text{nom}}+1}\zeta_{tk}(t_j)\Delta\lambda_j), \\ z_{m_f}(t_{j+1}) &= z_{m_f}(t_j) + \frac{\partial z_{m_f}}{\partial r_f}(t_j)\Delta r_{fj} + \frac{\partial z_{m_f}}{\partial r_{\text{mod}}}(t_j) r_{\text{cycl}}(t_j)\Delta\lambda_j \\ &\quad + (r_f(t_j) - r_{\text{mod}}(t_j)\frac{1}{K_{m\,\text{nom}}+1}\zeta_{tk}(t_j)\Delta\lambda_j), \end{aligned} \tag{13}$$

Here

$$\Delta r_{oj} = \int\limits_{t_j}^{t_j+\delta t} f_o(r_o, \alpha_{K_m}, \alpha_R) d\tau, \quad \Delta r_{fj} = \int\limits_{t_j}^{t_j+\delta t} f_f(r_f, \alpha_{K_m}, \alpha_R) d\tau,$$

where δt is transient time interval in object (4) with respect to coordinates r_o, r_f during abrupt change of α_{K_m} during implementation of control input $K_m(t)$ at time t_i.

For linearized engine equations under constant thrust mode, the values of propellant component flow increments due to changes in ratio coefficient K_m can be determined with the following expression [5]:

$$\Delta r_{oj} = \frac{\delta r_o(t_j)}{\delta K_m}\Delta K_{mj}, \quad \Delta r_{fj} = \frac{\delta r_f(t_j)}{\delta K_m}\Delta K_{mj}.$$

Let us rephrase the original terminal control problem. Instead of finding control $u(t)$ in the class of piecewise-constant functions, we search for the discrete sequence of coordinate increments $\vartheta(t), K_m(t), \lambda(t)$ at time points t_i, t_j.

Based on difference equations (11)–(13), we define algorithms for forming control input vector $\Delta u = (\Delta\vartheta_i, \Delta K_{mj}, \Delta\lambda_j)$ functions of the predicted boundary condition residuals.

The main disturbance in the terminal problem considered is the unknown initial conditions for the equations of the coordinates of the object (1), (4). The ability to counteract these disturbances when controlling the regions of the lower stage drop depends on the fact that the dimensions of the control vector are equal to the dimensions of the residual vector. When controlling the final stage, the dimensions of the boundary condition vector increase. In this case, to solve the terminal problem, it is necessary to choose the values of the control inputs for two discrete time points. In this case, the number of independent control inputs is larger than the dimensions of the residual vector. As a result of the analysis of possible options to form control inputs for two discrete time points, we adopted the following most obvious control algorithm. Consider a discrete time point t_j.

From the discrete equations (13) for the boundary condition residuals in terms of propellant consumption management, we determine the values of the control inputs $K_m(t_j)$, $\Delta\lambda(t_j)$. The control algorithm for the pitch angle with feedback based on predicted residual values $y_{pr}(t_{\text{command}}) - y_k$, $V_{ypr}(t_{\text{command}})$, which ensures the solution of the terminal problem under the given boundary conditions for the coordinates $y(t_k)$, $V_y(t_k) = 0$, is determined from equations (11), (12) for two discrete time points t_{i+1}, t_{i-p+1}. It should be noted that in the interval $[t_i, t_{i-p+1}]$, the residuals $y_{pr}(t_{\text{command}}) - y_k, V_{ypr}(t_{\text{command}})$ maintain their values unchanged.

The algorithm to control the pitch angle with the feedback predicted from discrepancies $y_{pr}(t_{\text{command}}) - y_k$, $V_{ypr}(t_{\text{command}})$ at discrete time points t_i, t_{i-p} is determined based on equation (12). It takes into account the value $\Delta\lambda(t_i)$, calculated in the propellant consumption control algorithm. The procedure to form this algorithm is described in [4]. In this case, the pitch angle at time t_{i-p} receives an increment $\Delta\vartheta_1$, while at time t_i it changes by an amount $\Delta\vartheta_2$.

The presence of parametric disturbances determines the errors in terminal control. We counter these disturbances by applying an iterative procedure to form the control vector $\Delta u = (\Delta\vartheta_i, \Delta K_{mj}, \Delta\lambda_j)$ with feedback on the vector of the residuals of the predicted boundary condition $z(t)$.

The main result of solving the problem considered of coordinated control of the center of mass motion and propellant consumption is the most complete utilization of available propellant reserves [6]. The essence of such coordinated control is as follows. Information about the current propellant mass is generated in accordance with (7), where $\lambda(t)$ is determined taking into account the measurements of the level sensors. We take this into account when predicting the discrepancies in the center of mass trajectory coordinates corresponding to the target of escape. In this case, equations (12) for $z_y(t_{j+1})$, $z_V(t_{j+1})$, $z_\delta(t_{j+1})$ include disturbance $\Delta\lambda_j$. By burning additional propellant, the final value of apparent velocity $W(t_{\text{command}})$ increases. The resulting error in the impact area is eliminated by varying the velocity in the neutral direction through additional pitch angle control. Note that the effectiveness of such control is maintained until the pitch angle approaches the value at which the maximum range of the spent stage is ensured.

Without taking into account the actual current value of the fuel mass in controlling the motion of the center of mass, the terminal time t_{command} is determined by the zero discrepancy in the coordinates of the trajectory. In this case, the effects of disturbing factors such as deviations in initial mass, propellant consumption, etc., on the trajectory that are countered by controlling the thrust vector up to the moment t_{command}, lead to significant unused propellant residues. The magnitude of these residues can reach 1% of the initial propellant mass.

In the propellant consumption control loop, significant random measurement errors occur when measuring the levels of propellant components in the tanks. As a result, even with error filtering, random control errors occur in the form of component residues at the moment t_k. To counteract these errors, we introduce safety reserve propellant components, which reduces the effectiveness of control. However, the implementation of coordinated terminal control for modern rocket boosters such as Angara and Soyuz-5 reduces unused propellant reserves by a factor of 3.

The principle of coordinated control of the center of mass movement and fuel consumption is implemented in the control algorithms of the Proton-M and Angara rocket boosters.

In foreign counterparts, terminal control of the center of mass movement by influencing the thrust vector and iterative procedures for feedback control based on predicted residuals was developed almost at the same time (at the end of the last century) as in the USSR and later in the Russian Federation. However, coordinated control of the center of mass movement and propellant consumption was not required. Presumably, because there were no strict constraints on the spent stages impact areas.

4. CONCLUSION

1. We consider the problem of synthesizing terminal control of the center of mass movement and propellant consumption for liquid rocket boosters. The control synthesis problem is limited by given boundary conditions, the fulfillment of which is a priority task.

When solving the problem, we assume that the system can be decomposed into interrelated processes of final-state control and object stabilization. Decomposition allows us to reveal the content of control processes in the terminal system. Terminal control is performed by specifying the object coordinate values maintained by the stabilization loop. Stabilization of the object relative to the given values is characterized by fast damping of the dynamics of transient processes. The derivative of the residuals in the decomposed system explicitly depends on the terminal control.

2. We solve the synthesis problem in the class of systems with the prediction of boundary condition residuals, which are vector functions of the current values of the object coordinates and time. We discretized the synthesis problem for control variations in the class of piecewise-constant functions. We obtain difference equations for the vector of predicted residuals. We determined algorithms to form the vector of control actions to change the pitch angle, the proportion of component consumption rates, and the controlled parameter of the object model as functions of the predicted residuals of the boundary condition based on the difference equations obtained.

3. The solution to the considered terminal problem is a jointly coordinated control of the center of mass movement and propellant consumption, ensuring the most complete use of available propellant reserves. The principle of coordinated control of the center of mass movement and fuel component consumption is implemented in the control algorithms of the Proton-M rocket booster and the Angara rocket booster family.

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